Delta Modulation

Final Report

January 1, 1972 - December 31, 1972

National Aeronautics and Space Administration

MSC - Houston

under

NASA Grant NGR 33-013-063

COMMUNICATIONS SYSTEMS LABORATORY
DEPARTMENT OF ELECTRICAL ENGINEERING

THE CITY COLLEGE OF
THE CITY UNIVERSITY of NEW YORK

Donald L. Schilling
Professor of Electrical Engineering
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Introduction

This final report summarizes the research sponsored by the National Aeronautics and Space Administration under NASA Grant NGR 33-013-063 for the period January 1, 1972 through December 31, 1972. The research supported by this grant encompasses the problems of source encoding using delta modulation.

Part I of this report discusses the final algorithm employed in our design of an adaptive delta modulation. The algorithm is entremely inexpensive to implement and features extremely desirable characteristics such as a 40 dB dynamic range and a 90% non-redundant word intelligibility at a bit rate of 9.6 kb/s.

Part II of this report presents a new concept in delta modulation design, The Nth-Order Delta Modulator. Adaptive delta modulators are used to increase the dynamic range of a delta modulator, however they do not increase the SNR of the delta modulator. The Nth-Order Delta Modulator increases the maximum signal-to-noise ratio of a delta modulator but not its dynamic range. It is shown in this section that a 2nd-Order Delta Modulator, one which employs two delta modulators in "cascade", results in a 15 dB output SNR improvement over a linear delta modulator operating at 27 dB. The theoretical results presented here have been verified by simulating the systems on a hybrid analog-digital computer. The results are also presented.

Two New Areas of Research are discussed in Part III. The first is the Decomposition of Voice. It is expected that this particular decomposing technique will enable us to transmit voice, digitally, at a 2.4 kb/s rate, with high intelligibility and recognizeability.

The second new area of research is concerned with the Source Encoding of Video Signals. It is expected that adaptive delta modulation using the Song Algorithm * can be used.

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* Song, Garodnick and Schilling, "A Robust Delta Modulator", IEEE Transactions on Communications, December 1971, Part II.

The results presented in this report represent a significant step forward in the design of delta modulators. The research supported by this grant has resulted in 1 PhD dissertation (Part IV) and in the Publication of Several Papers (Part V).

Participating in this program were:

Drs. J. Garodnick and I. Paz

and

Messrs., J. Frank, J. LoCicero, M. Steckman, E. Feria and L. Maldonado.

Summary of Results

1. An All-Digital Adaptive Delta Modulator

An all-digital adaptive delta modulator was designed and constructed at the Communications Systems Laboratory, using integrated circuits and given to NASA-Houston. Tests performed by NASA-Houston indicate a word intelligibility of 88% at a bit rate of 9.6 kb/s and a word intelligibility of 98% at a bit rate of 19.2 kb/s.

The delta modulator also has a 40 dB dynamic range as contrasted to a 10 dB dynamic range available with most delta modulators. This is extremely important because it enables one to understand a soft speaker as well as a loud speaker. Furthermore, since the device is digital the dynamic range is directly proportional to the number of bits employed in the internal arithmetic of the device. We are using 10 bits; the dynamic range can be increased by 6 dB for each extra bit added.

2. The Nth-Order Delta Modulator

An Nth-order DM was designed at the Communications Systems Laboratory and simulation tests made to verify its performance.

The Nth-order DM time division multiplexes the same digital DM, N times to encode not only the signal, but its error, the error in estimating the error, etc. As a result, the received signal has a significantly smaller error and therefore a higher SNR.

The research to date has centered on developing the theory and verifying these results using hybrid computer simulation. It has been shown, that for a 2nd-order DM a SNR increase of 8 dB occurs when the ordinary DM has a SNR of 22 dB, and a 15 dB SNR increase occurs when the ordinary DM has a SNR of 30 dB.

We are now connecting the 2nd-order DM to our all-digital adaptive DM. Our expectation is to obtain an increase in SNR with a wide dynamic range.

3. Decomposition of Voice

We are attempting to decompose voice into its AM and FM counterparts. Since FM is a bandwidth expansion process, demodulating the FM part should result in a significant bandwidth compression.

We are currently designing the circuits needed to decompose and demodulate the signal.

4. Video Encoding using Deltamodulation

Since we have succeeded in encoding voice using DM we are now attempting to encode video signals. The DM algorithm employed for video allows the step-size of the DM to increase or decrease exponentially rather than linearly (which is employed for voice). Also, the 40 dB dynamic range of our DM will enable encoding of signals which would be lost by other types of systems.

We are currently calibrating our video systems and expect to have results before the end of 1973.

Part I The Schilling All-Digital Adaptive Delta Modulator

An all digital adaptive delta modulator (ADM) was constructed at the Communications Systems Laboratory of the Department of Electrical Engineering of the City College of New York. This unit was given to NASA-Houston who tested it and compared it with other units made available by private industry. To date, the conclusions are that a 90% word intelligibility is possible with our device, at a bit rate of 9.6 kb/s and a 98% word intelligibility results at 19.2 kb/s; also, NASA found that the dynamic range of our unit was significantly greater than the dynamic range of each of the other units tested.

The algorithm employed in the design of this unit was significantly less complicated than the original Song DM which was constructed in 1970. This resulted in a low cost device; our unit cost \$1000 in parts.

ALGORITHM

The algorithm is

$$x(k+1) = x(k) + S(k+1) \tag{1}$$

where $x(k)$ is the estimate of the incoming analog signal at the sample time k/f_s where f_s is the sampling rate, and $S(k+1)$ is the new step-size at time $(k+1)/f_s$.

Equation 1 is a basic equation true for all delta modulators. For example for the linear DM, $S(k+1) = \pm S_o$, i.e., the magnitude of the step-size is fixed (at S_o). In our system,

$$S(k+1) = | S(k) | e_k + S_o e_{k-1} \tag{2}$$

where e_k is the <u>sign</u> of the error which occurs at time k/f_s and S_o is the voltage associated with the minimum step-size. In our system, which employed 10-bit internal arithmetic, $S_o = 10$ mV.

We note that if m_k is the signal value at time k/f_s, then

$$e_k = \text{sgn}\ (m_k - x_k) \tag{3}$$

Equations 2 and 3 are the "heart" of our algorithm. We see from these equations that the new step-size $S(k+1)$ differs, in <u>magnitude</u>, from the old step-size by $\pm S_o$ volts. For example, if $| S(k) | = 10\ S_o$

and $e_k = +1$ while $e_{k-1} = -1$ then $|S(k+1)| = 9\ S_o$. On the other hand if $e_k = +1$ and $e_{k-1} = +1$, $|S(k+1)| = 11\ S_o$.

BLOCK DIAGRAM

The complete block diagram of the ADM encoder is shown in Fig 1. The signal m(t) is sampled every $1/f_s$ to form m(k+1). This sample is compared to the new estimate x(k+1) to form the sign of the error e_{k+1} (see Eq 3). Next e_k and e_{k-1} are obtained using delay flip-flops. The new step-size S(k+1) is formed by feeding the old step-size S(k) into block F with e_k. This device merely alters the sign of S(k) if the sign of S(k) and e_k differ. This is accomplished using exclusive-or gates. Equation 2 is generated using the adder. Knowing S(k+1) and "remembering" the previous estimate x(k), we add to form x(k+1).

The decoder is merely the feedback network between e_{k+1} and x_{k+1}.

It is interesting to note that the linear DM, which is characterized by the equation

$$x(k+1) = x(k) + S_o\ e_k \tag{4}$$

is constructed by connecting e_k directly to the Σ_1 input marked S(k+1). When this is done, e_k is no longer merely a sign bit, but now also includes the least significant bit which represents S_o.

OSCILLATIONS

If the input signal m(t) is constant, the ADM reaches a steady state condition and oscillates about the quiescent voltage with a fundamental frequency of $f_s/4$.

Fig 2 illustrates the condition of oscillation. Unfortunately the amplitude of the oscillations depends on the step-size at the time of

the initial overshoot. These oscillations can be eliminated by adding noise to the signal m(t) where the noise amplitude is S_o. In practice, however, the oscillation is not bothersome and does not deteriorate the quality of the sound. Thus, the addition of the noise source is not employed.

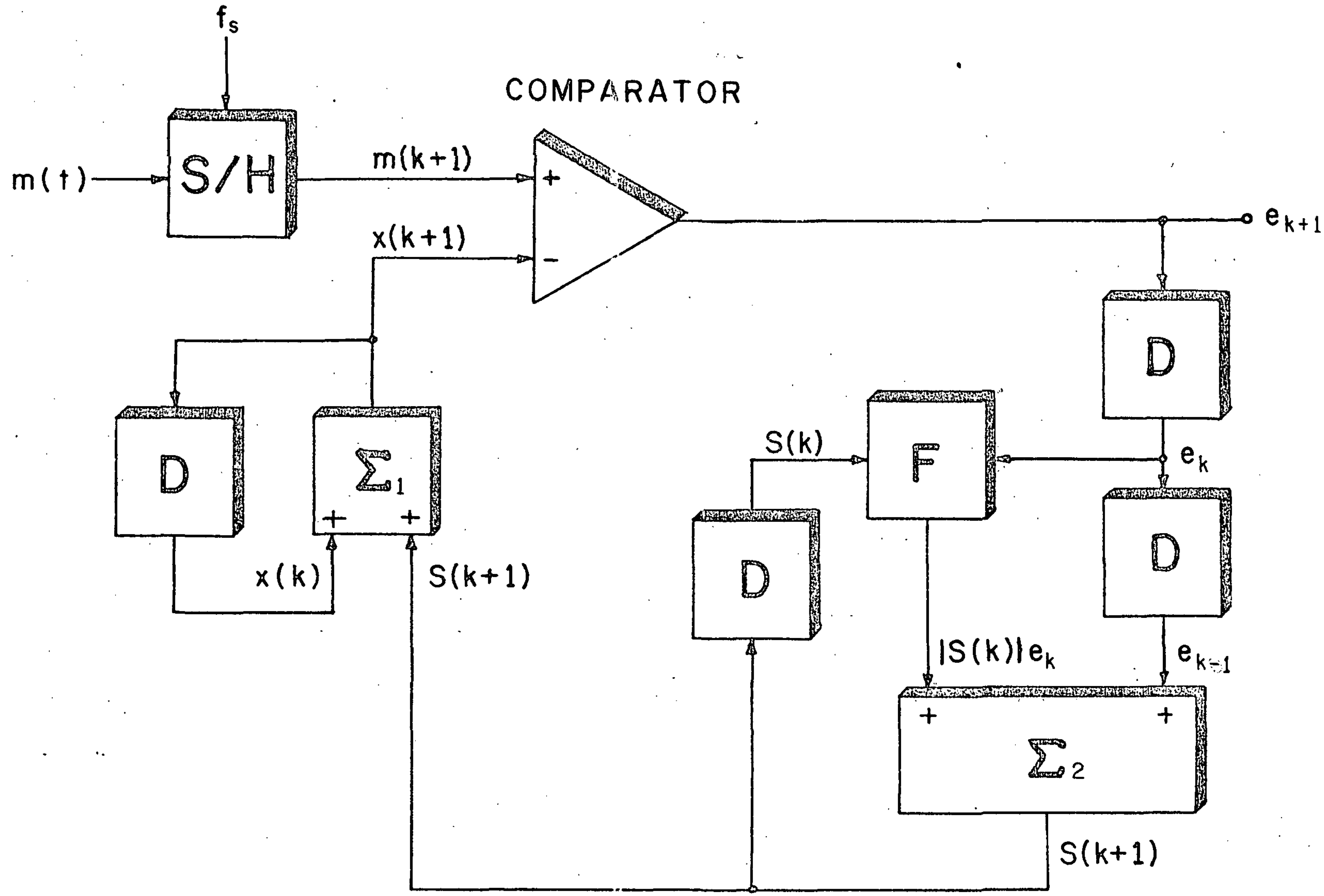

Figure 1 The Schilling Adaptive Delta Modulator

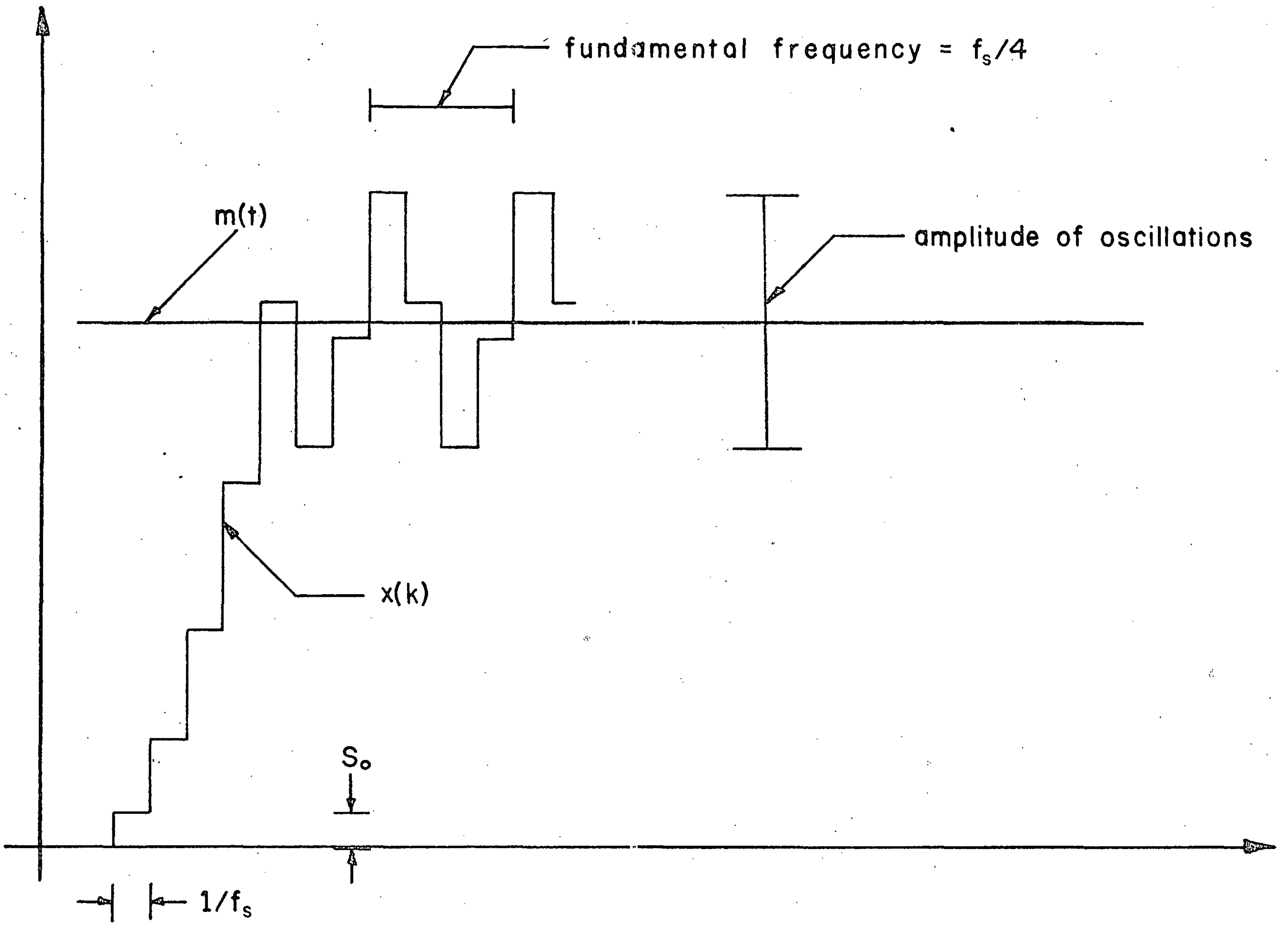

Figure 2 Waveforms Showing Oscillations in x(k) when m(t) is Constant

II. The N-th Order Delta Modulator

INTRODUCTION

When PCM is used to encode a signal, the decoded estimate differs from the original signal, as a result of quantization noise. When a signal is encoded using delta modulation, the estimate x(t) again differs from the original signal m(t) by quantization noise generated by the difference d(t), where

$$d(t) = m(t) - x(t) \tag{1a}$$

(Note that e(t) = sgn (d(t))). Thus the signal m(t) is

$$m(t) = x(t) + d(t) \tag{1b}$$

If the receiver knew d(t) as well as x(t) then m(t) could be reconstructed without error. In a 2nd order DM an estimate of d(t) is transmitted along with x(t) by delta modulating d(t) using a second DM. Let us call the estimate of d(t), $\hat{d}(t)$ and its error $d_1(t)$. Then

$$d(t) = \hat{d}(t) + d_1(t) \tag{2}$$

At the receiver, the reconstructed signal estimate is now (called) $\hat{m}(t)$ which is

$$\hat{m}(t) = x(t) + \hat{d}(t) = x(t) + d(t) + d_1(t) = m(t) + d_1(t) \tag{3}$$

Thus, the error present at the receiver is the error present in the second system, $d_1(t)$.

If the error in the first DM is small and the error in the second DM is small, then the error in the receiver is extremely small. This concept can, of course, be extended to N delta modulators, wherein the final error is $d_{N-1}(t)$

Characteristics of $d_{N-1}(t)$

While m(t) is bandlimited to the frequency range 0 to f_M, the difference $d_N(t)$ is extremely wide band and extends beyond the sampling

frequency f_s. Since the receiver is interested in the portion of $\hat{d}(t)$ which is located within the signal band, the remainder is filtered out; we prefilter d(t) prior to DM encoding it. This is shown, for a 2nd order DM in Fig 1.

Figure 1 shows that the difference d(k+1) is digitally filtered before being applied to the second DM. In addition, although it is not shown explicitly, d(k+1) is also amplified. The reason is that the inband power level of d(k+1) is much less than m(k+1). To permit both, or all N delta modulators to be identical, the power in $d_f(k+1)$ must be the same as in the signal m(k+1).

It is interesting to note that as a result of the filtering process, the amplitude distribution of $d_f(d+1)$ tends toward a gaussian process.

Sampling rate

In order to compare an Nth order DM and an ordinary DM (N=1) we must require that the transmitted bit rate by constant independent of N. Thus, the sampling rate for N=1, $R_1 = f_s$. When N=2, both e_{k+1} and $e_{1,k+1}$ (see Fig 1) are transmitted similtaneously. For a transmission rate of f_s b/s, each of the delta modulators operate at a rate $R_2 = f_s/2$. Accordingly, for an Nth order DM each DM operates at a bit rate f_s/N.

Signal-to-Noise Ratio of an Nth Order DM

The output SNR of a linear DM is a function of two parameters, the bandwidth expansion factor $B = f_s/2f_M$ and the ratio of the input signal power to the square of the step size, $\gamma = P_i/S_O^2$ where P_i is the input signal power. A typical curve of output SNR vs γ is shown in Fig 2 for a given B.

We note that a γ exists which produces a maximum output SNR. In an adaptive DM, the step size S_O is adjusted to insure that for all P_i, in a specified range, the output SNR is a maximum.

The variation of the maximum SNR_Q with B has been found by Frank to be

$$\text{max. } SNR_Q = C \cdot B^{2.4} \tag{4}$$

where C is a constant which depends on the signal characteristics. Actually, Eq 4 is approximately correct for any output SNR_Q and not just the maximum value.

For an Nth order DM, the output SNR_Q of each DM is then,

$$(SNR_Q)_n = C \cdot B^{2.4} \frac{1}{N^{2.4}} \tag{5}$$

where n is any of the delta modulators n = 1, 2, . . ., N. Note that the output SNR is decreased as a result of decreasing the sampling rate of each DM.

To obtain the overall output SNR_Q of an Nth order DM let us first consider the SNR of a 2nd order DM. Equation (3) indicates that the noise is $d_1(t)$. This noise will be filtered before its power is measured. Let us call this filtered noise $d_{f,1}$. Then, for a 2nd order DM

$$SNR_{Q,2} = \frac{\overline{m^2}}{\overline{d^2_{f,1}}} = \frac{\overline{m^2}}{\overline{d^2_f}} \cdot \frac{\overline{d^2_f}}{\overline{d^2_{f,1}}} \tag{6}$$

$\overline{m^2}/\overline{d_f^2}$ is the output SNR of the first DM and $\overline{d_f^2}/\overline{d^2_{f,1}}$ is the output SNR of the second DM, since d_f is the output to the second DM and the power of the estimate $\hat{d}_f$ is comparable to the input power d_f.

Since both DMs are identical with the same input characteriestics, the output SNR of each DM is the same. Hence

$$SNR_{Q,2} = \left[SNR_{Q,1} \right]^2 \tag{7}$$

where $SNR_{Q,1}$ is the output signal to noise ratio of a single DM.

Similarly, for an Nth order DM,

$$SNR_{Q,N} = \left[SNR_{Q,1} \right]^N \tag{8}$$

Combining Eqs 5 and 8 we have

$$SNR_{Q,N} = \left[CB^{2.4} \right]^N N^{-2.4N} \tag{9}$$

We note that $CB^{2.4}$ is the SNR_Q of an ordinary delta modulator (N=1). We give this SNR the symbol α and we wish to determine the increase in $SNR_{Q,N}$ over α. To do this, we differentiate with respect to N. This yields

$$N_{OPT} = e^{-1} \alpha^{1/2.4} \tag{10}$$

Equation 10 shows that the optimum value of N is a function of the output SNR of the ordinary DM, α. We, however, show below that this dependence is not critical.

The maximum SNR_Q of the Nth order DM is then

$$\begin{aligned} SNR_{Q,N} &= \alpha^{N_{OPT}} N_{OPT}^{-2.4 N_{OPT}} = \left(\frac{\alpha}{N_{OPT}^{2.4}}\right)^{N_{OPT}} \\ &= e^{2.4 \left[e^{-1} \alpha^{1/2.4} \right]} \end{aligned} \tag{11}$$

or

$$\left[SNR_{Q,N} \right]_{dB} \simeq 3.8\, \alpha^{1/2.4} \tag{12}$$

A plot of the maximum SNR of an optimum Nth order DM as a function of α is shown in Fig 3. Note that is α increases, the $SNR_{Q,N}$ increases dramatically. However, in a practical system N is fixed and cannot vary with α.

Figure 4 shows the SNR of a Nth order DM as a function of the SNR of an ordinary DM. Note that the plot is made holding N constant. It should also be noted that improvement results only when the curves for N=2 and 4 lie above the curve for N=1. This figure was obtained by plotting Eq 9.

Experimental Results

The ordinary linear DM and a 2nd order DM were simulated on a hybrid computer. In analog input signal was applied to each system.

The input signal was bandlimited from 0 to 3.1kHz using a 4 pole filter and had a normal amplitude distribution.

A critical part of the design of the second order DM was the coupling filter used between stages. This is seen if we remember that the purpose of the filter is to eliminate high frequency components and thereby maintaining a high f_s/f_m ratio and hence a high SNR for the second DM. However, in the receiver, the samples decoded to form analog signals are then added. The analog signal estimating the error, $\hat{d}_f(t)$, is delayed by the filter with respect to x(t). A deemphasis filter must therefore be used to remove the delay distortion.

Several deemphasis-type techniques exist. One is to filter x(t) before adding $\hat{d}_f(t)$ using the preemphasis filter. Another is to use a non-recursive filter which provides linear delay. This delay is easily removed in the receiver by delaying x(t).

In this research the filtering between the first and second DM was performed using a 6-pole Butterworth filter and the delay was cancelled in the receiver.

The results obtained are shown in Fig 5. In this figure the input SNR is unimportant since a change in the step-size would produce a shift along the abscissa. What is important is the 8 dB maximum SNR improvement obtained using the 2nd order DM. This result verifies Fig 4 which predicts a 30 dB SNR when the SNR of the ordinary DM is 22 dB.

It is interesting to observe that the maximum SNR of the 2nd order DM occurs at a lower input SNR. This is due to the fact that each of the two delta modulators are sampled at a 50 kHz rate and therefore slope overloading occurs sooner.

Conclusions

The results shown here indicate that substantial improvements can be obtained using an Nth order DM.

Furthermore, if the Nth order DM is employed with our adaptive digital delta modulator an increase in SNR should result over the full dynamic range of the adaptive DM. Also, since the adaptive DM is digital, time division multiplexing can be employed so that a single DM can be used rather than N delta modulators. It should be noted that the digital filter can then also be multiplexed so that a single filter is required. This application is currently being pursued.

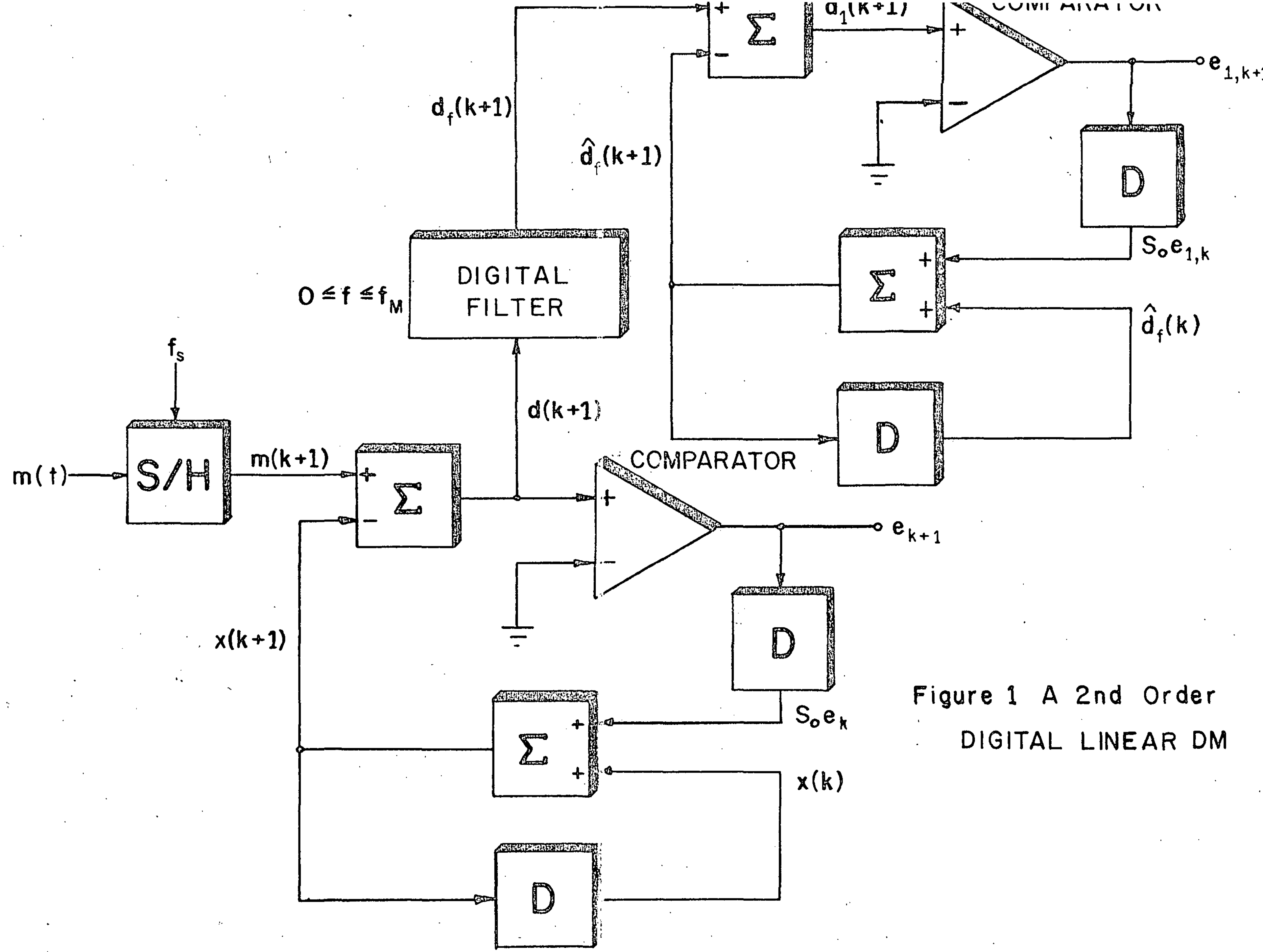

Figure 1 A 2nd Order
DIGITAL LINEAR DM

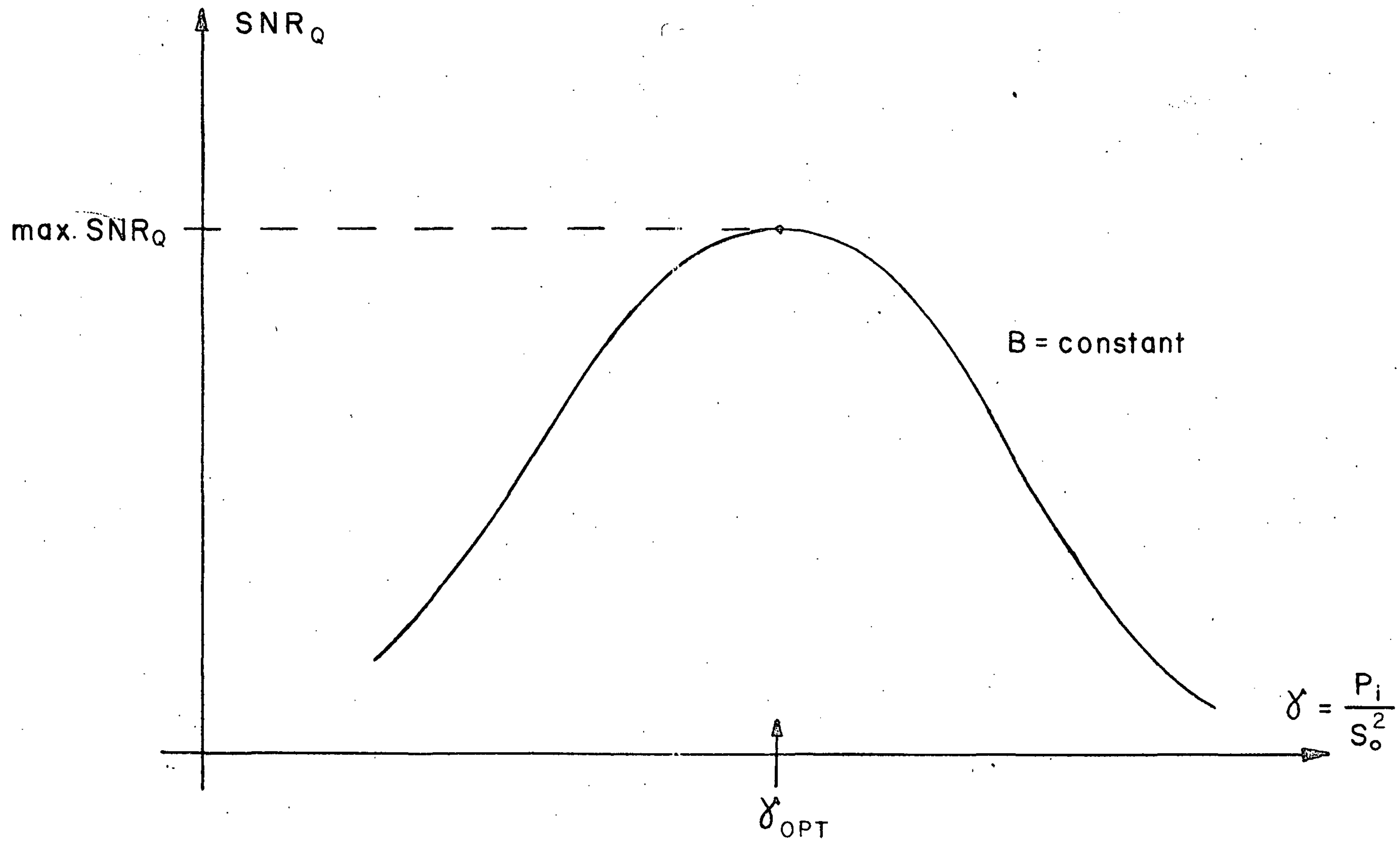

Figure 2 Plot of SNR_Q vs. γ for B = constant

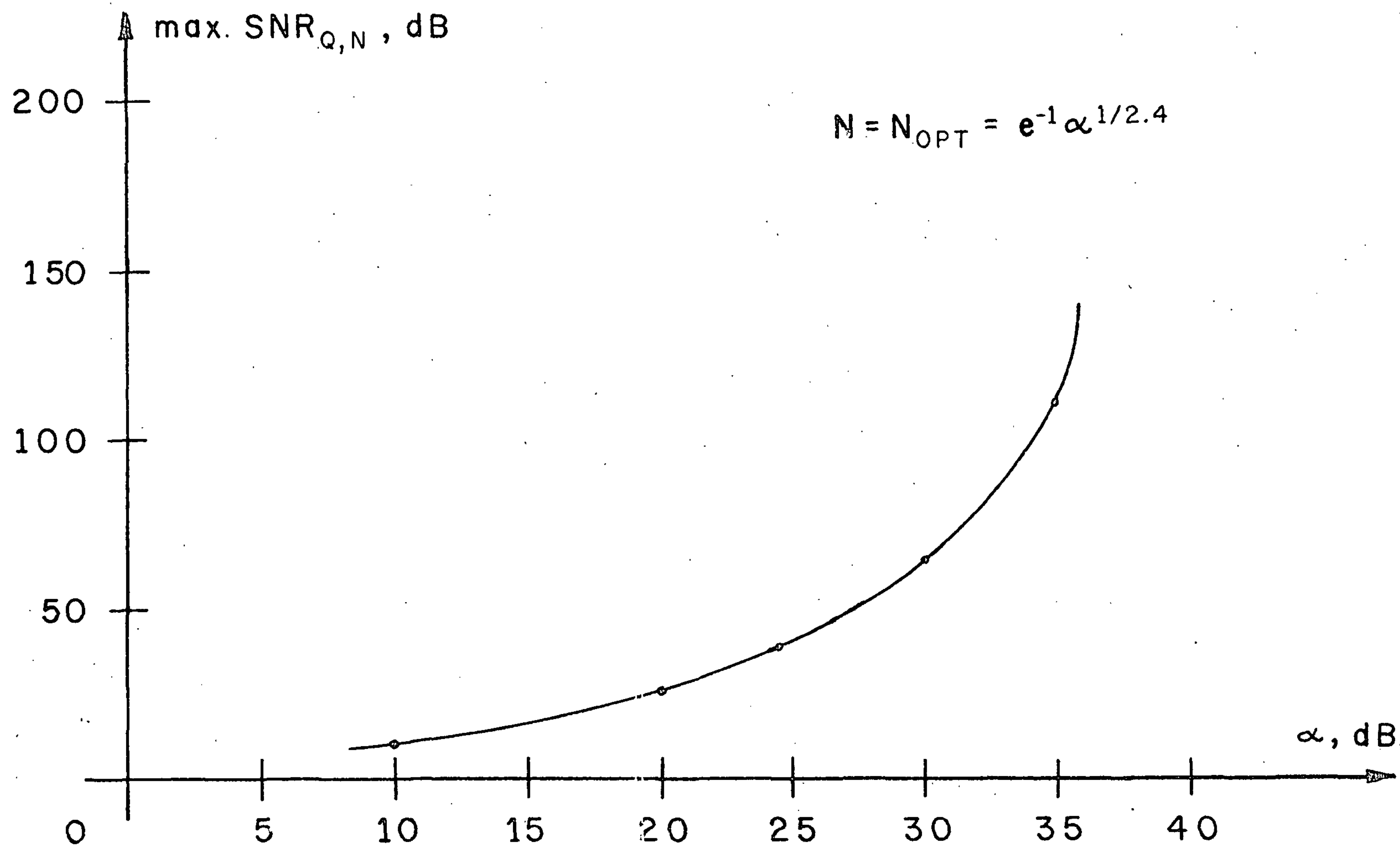

Figure 3 Maximum Output $SNR_{Q,N}$ of the Nth Order DM as a Function of the Output $SNR_{Q,1}$ of an Ordinary DM, when $N = N_{OPT}$

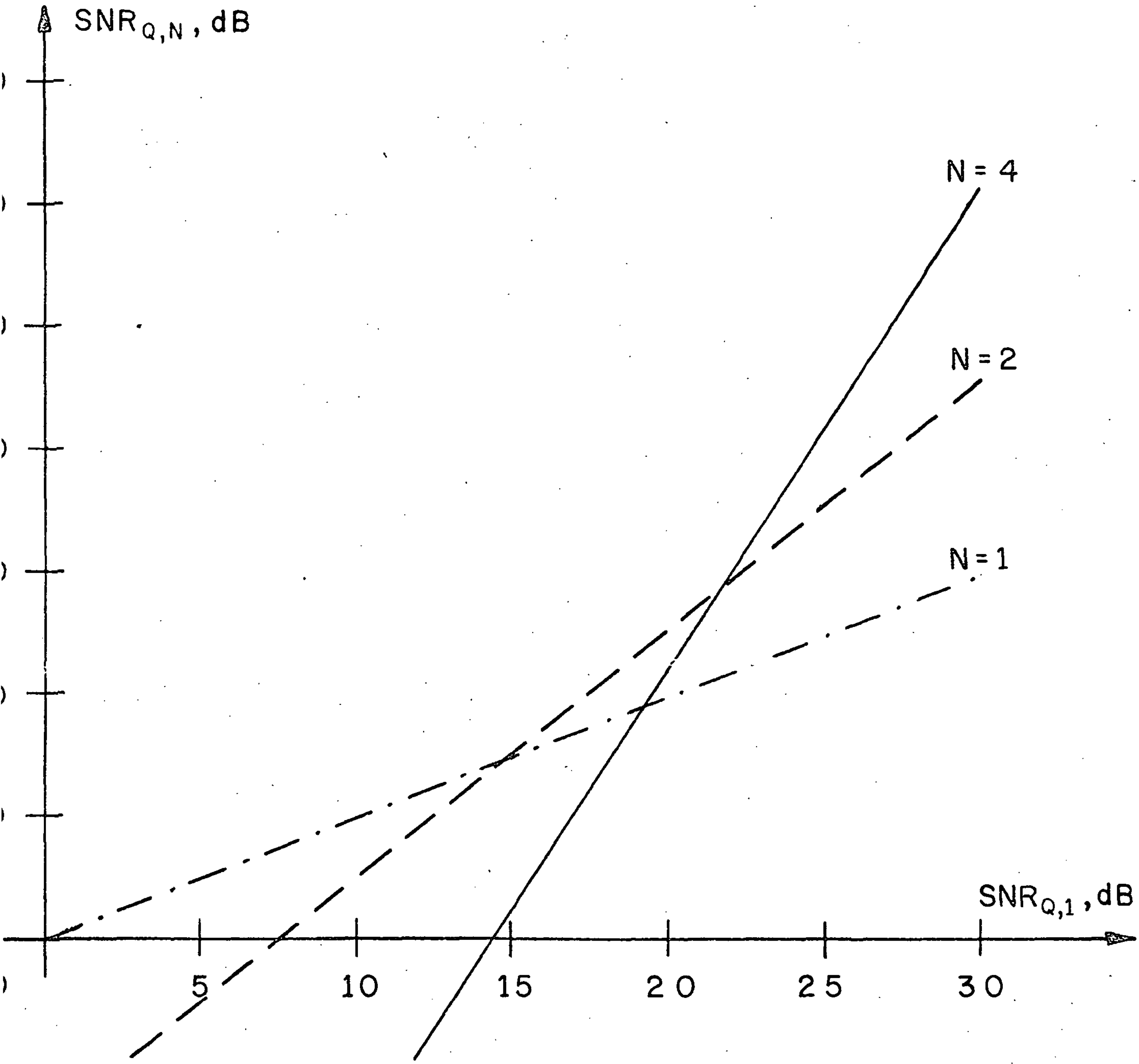

Figure 4 SNR Variation for a Fixed N

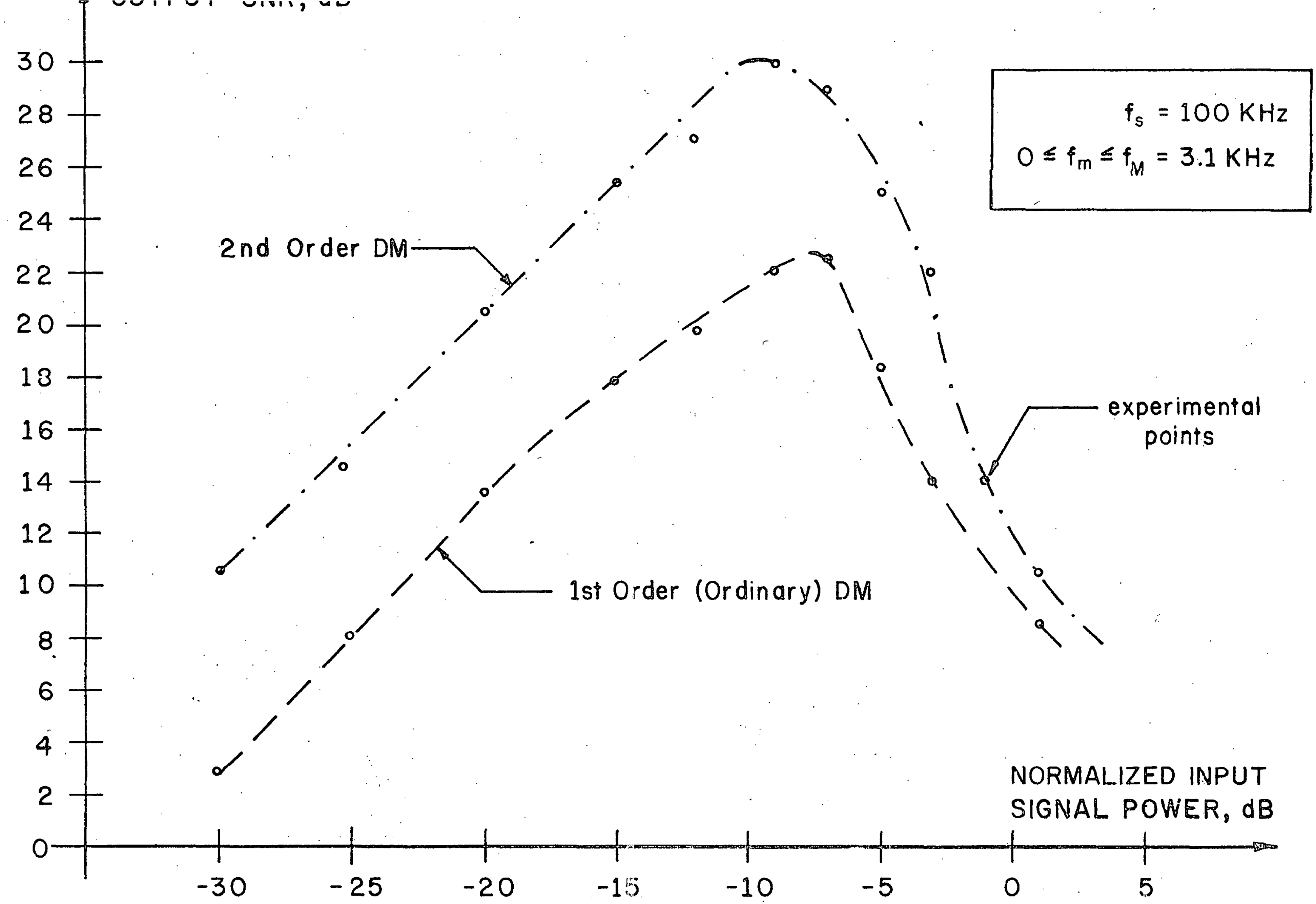

Figure 5 Experimental Comparison of Ordinary and 2nd Order DM

III. New Areas of Research

1. Voice Decomposition

Any voltage v(t), whether a wideband or a narrowband process, can be represented as

$$v(t) = R(t) \cos \phi (t) \tag{1}$$

When v(t) is a voice signal the reason for using the representation of Eq 1 is as follows: We know from experience that FM is a bandwidth expansion process, thus, if ϕ (t) is a narrowband process, cos ϕ (t) is a wideband process.

If v(t) represents a voice signal then cos ϕ (t) is wideband. Rather than encoding v(t) we propose to determine R(t) and $\dot{\phi}$ (t) and encode these narrowband processes.

To date we have successfully extracted R(t) and found it to have approximately a 100 Hz bandwidth. Frequency demodulation of cos ϕ (t) is more difficult to accomplish since cos ϕ(t) is a wideband process. Digital demodulation techniques are currently being applied to this process.

If we are successful and find that $\dot{\phi}$ (t) require a bandwidth of several hundred Hz we will have successfully decomposed voice into narrowband components which can then each be digitally encoded for transmission. The result is transmission of voice using significantly less bandwidth than is now possible.

2. Video Encoding

The Song adaptive delta modulator($\alpha = 1$, $\beta = \frac{1}{2}$) can be used to encode video signals as a result of the rapid (exponential) rise and fall times.

The DM constructed in Communications Systems Laboratory can operate at bit rates up to 1MHz. Thus, to study video encoding we must employ a slow-scan video system. We are doing this by using a flying-spot scanner with an adjustable scan rate.

Before making any quantitative studies we are first calibrating our system. Further, we are looking into the use of quantitative techniques such as SNR to describe contour jitter and resolution.

The encoding of video signal will occur after preemphasizing the signal. Several preemphasis networks will be employed. The object of using preemphasis is to increase the rise and fall times of the pulses which make up the video signal. This tends to keep the DM out of the slope overload region.

IV Doctoral Dissertation

1973, Joseph Frank, "Deltamodulation"

V. Papers Published and Presented

1. "Distortion in the FMFB", IEEE Transactions on Communications April 1972
2. "Compression of Transmission Bandwidth Requirements for a Certain Class of Band-Limited Functions", IEEE Transactions on Communications, April 1972
3. "Gaussian and Click Noise in an FM/FM Multiplex Receiver", IEEE Transactions on Communications, June 1972, and ICC - 6/72
4. "Theory of Operation and Design of an All-Digital FM Discriminator", IEEE Transactions on Communications, December 1972 and ICC - 6/72
5. "All Digital PLL for FM Demodulation, ICC - 6/72
6. "An Adaptive DM for Speech and Video Processing, ICC - 6/72

THE CITY COLLEGE of THE CITY UNIVERSITY of NEW YORK

Convent Avenue at 138th Street, New York, New York 10031

Telephone: (212) 281-0470; (212) 368-1444

The City College Research Foundation

The City College Office—Research Foundation of The City University of New York